# Burlington County Institute of Technology

Medford Campus | Westampton Campus

**Precalculus**

Department: Mathematics
Credits: 5
Revised: August 2023
Board Approval Date: August, 2023

# Course Description

Precalculus is an honors course that provides challenging and in-depth study of discrete topics in advanced algebra and trigonometry.  This course is designed for students who have a strong background in Algebra 2 and are highly motivated to explore the subject in greater detail.

Students will investigate theoretical, numerical, graphical, and spatial topics upon which to build their study of advanced mathematics. Precalculus provides the background for mathematical concepts, problems, issues, and techniques that appear in the study of calculus, including but not limited to: functions, trigonometry, vectors, parametric equations, polar coordinates, complex numbers, conic sections, exponential functions, and logarithmic functions. The use of technology is infused in this course to gather, analyze, and communicate mathematical information.

Precalculus is an advanced mathematics course that serves as a bridge between algebra, geometry, and calculus. This course is designed to help students develop a strong foundation in the fundamental concepts of mathematics and prepare them for more advanced courses in mathematics and science.
In precalculus, students will explore functions, their properties, and their graphs, as well as trigonometry, complex numbers, and vectors. This course will also cover topics such as sequences and series, exponential and logarithmic functions, and conic sections.

Students will be required to demonstrate a high level of proficiency in algebraic manipulation, problem-solving, and analytical thinking. Throughout the course, students will use technology, such as graphing calculators and computer software, to enhance their understanding of mathematical concepts and to solve complex problems.

Upon successful completion of this course, students will have a strong foundation in mathematics and will be prepared to tackle the challenges of higher-level mathematics courses. Students will also develop problem-solving skills that are applicable in a wide range of academic and professional settings.

# Table of Contents

# Pacing Guide

| Unit | Standards | Days |
|---|---|---|
| Unit 1:  Functions from a Calculus Perspective | F-IF: 7d<br>F-BF: 1c, 4b<br>A-APR: 7<br>A-REI: 8, 9 | 10 |
| Unit 2:  Trigonometric Functions | G-SRT: 6, 7, 8, 10, 11<br>F-TF: 3, 4, 6, 7 | 26 |
| Unit 3:  Analytic Trigonometry | F-TF: 8, 9 | 6.5 |
| Unit 4:  Vectors and Parametric Equations | N-VM: 1, 2, 3, 4a, 4b, 4c, 5a, 5b | 10 |
| Unit 5:  Polar Coordinates and Complex Numbers | N-CN: 3, 4, 5, 6, 8 | 11 |
| Unit 6: Conic Sections | G-GPE: 3 | 7 |
| Unit 7: Sequences and Series | A-SSE: 4<br>A-APR: 5<br>F-BF: 2 | 6 |

# Curriculum Maps

| Unit 1: Functions from a Calculus Perspective | |
|---|---|
| Desired Outcomes | |
| **Established Goals: NJSLS**<br><br>● Analyze functions using different representations (F.IF.7d)<br>● Build a function that models a relationship between two quantities (F.BF.1c).<br>● Find inverse functions (F.BF.4b).<br>● Rewrite rational expressions (APR.7).<br>● Solve systems of equations (A.REI.8, A.REI.9)<br><br>**NJSLS Mathematical Practices**<br>1. Make sense of problems and persevere in solving them.<br>2. Reason abstractly and quantitatively.<br>3. Construct viable arguments and critique the reasoning of others.<br>4. Model with mathematics.<br>5. Use appropriate tools strategically.<br>6. Attend to precision.<br>7. Look for and make use of structure.<br>8. Look for and express regularity in repeated reasoning. | |
| **Enduring Understandings:**<br><br>○ Mathematics is a language consisting of symbols and rules.<br>○ There is more than one way to represent expressions.<br>○ Characteristics of functions can be used to model | **Essential Questions:**<br><br>○ How can mathematical ideas be represented?<br>○ How are symbols useful in mathematics?<br>○ Based on a given function, how can the characteristics of the graph of the function be determined? |

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| real life situations. | ○ What characteristics of functions can help you analyze real world situations? |
| **Students will know:**<br><br>○ A graph conveys information about the function or relation it represents. Important characteristics of a graph include the following: domain, range, y-intercept, zeros, even and odd functions, line and point symmetry.<br>○ The limit of a function is the y-value that is approached from the left and from the right of a specific x-value. Limits give information about continuity. Different types of discontinuity include: infinite, jump, removable.<br>○ Graphs provide a quick way to understand the relationship between different variables. This includes finding the maxima and minima. These are the points where the relationship between the variables changes from increasing to decreasing or vice versa.<br>○ The average rate of change between any two points on a graph is the slope of the line through those points.<br>○ The difference between an even and odd function<br>○ The end behavior of functions<br>○ Behaviors of graphs can reveal various patterns.<br>○ Key vocabulary: zeros, roots, line symmetry, point symmetry, even function, odd function, continuous, limit, discontinuous, infinite, jump, point, removable discontinuity, nonremovable discontinuity, end behavior, increasing, decreasing, constant, maximum, minimum, | **Students will be able to:**<br><br>○ Identify even and odd functions.<br>○ Use limits to determine the continuity of a function.<br>○ Use limits to describe the end behavior of functions.<br>○ Find intervals on which functions are increasing, constant, or decreasing.<br>○ Determine the average rate of change of a function.<br>○ Graph rational functions and determine vertical, horizontal, and slant asymptotes.<br>○ Determine roots of polynomial equations.<br>○ Solve rational and radical equations and inequalities. |

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| extrema, average rate of change, secant line, zeros, asymptote, polynomial, leading coefficient, root, imaginary number, complex numbers, rational equation, rational inequality, radical equations, extraneous solutions, radical inequalities | |

## Assessment Evidence

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| **Suggested Performance Task:**<br><br>○ Have students choose a function that interests them. For example, students interested in music might find statistics on time between their favorite band's record releases and the number of records sold. Once students have decided which function to explore, have them collect data to write a function rule. Have students graph the function and specify the domain, range, extrema, y-intercepts, and zeros. In each case, have students write about the practical implications of these values. | **Required District/State Assessments:**<br><br>○ Unit Assessment<br>○ SGO Assessments<br><br>**Suggested Formative/Summative Assessments:**<br><br>○ Describe Learning Vertically<br>○ Identify Key Building Blocks<br>○ Make Connections (between and among key building blocks)<br>○ Short/Extended Constructed Response Items<br>○ Multiple-Choice Items (where multiple answer choices may be correct)<br>○ Drag and Drop Items<br>○ Use of Equation Editor<br>○ Quizzes<br>○ Journal Entries/Reflections/Quick-Writes<br>○ Accountable talk<br>○ Projects<br>○ Portfolio<br>○ Observation<br>○ Graphic Organizers/ Concept Mapping<br>○ Presentations<br>○ Role Playing |

- Teacher-Student and Student-Student Conferencing
- Homework

## Learning Plan

**Learning Activities:**

- Starter exercises
- Guided notes as necessary
- Formative assessments (QR codes, scavenger hunt, interactive exercises, exit ticket, etc.)

## Related Standards

**Interdisciplinary connections**

- ELA Literacy Connection (NJSLS W.11-12.1): Publishing Cost Task

A publishing company estimates that the average cost (in dollars) for one copy of a new scenic calendar it plans to produce can be approximated by the function

$$C(x) = \frac{2.25x + 275}{x}$$

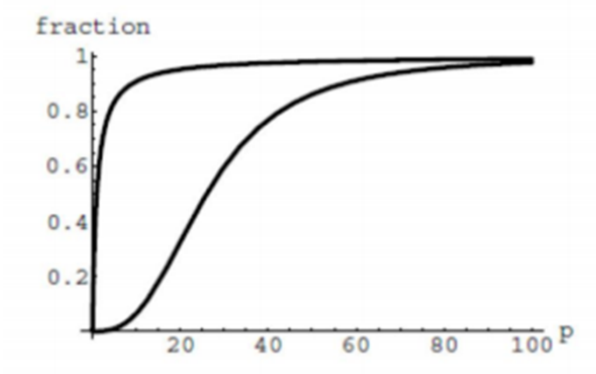
Where $x$ is the number of calendars printed.

a. Find the average cost per calendar when the company prints 100 calendars.
b. Identify the domain and range of this function.
c. After analyzing the function, Alex said that this company should not be allowed to publish zero calendars. As a result, the company has no option to shut down and go out of business. Write an argument to support or reject Alex's conclusion.

- Science Connection (NJSLS: HS-LS1-2; HS-LS1-4): Myoglobin and Hemoglobin

Myoglobin and hemoglobin are oxygen-carrying molecules in the human body. Hemoglobin is found inside red blood cells, which flow from the lungs to the muscles through the bloodstream. Myoglobin is found in muscle cells. The function $Y = M(p) = \frac{P}{1+P}$ calculates the fraction of myoglobin saturated with oxygen at a given presure $p$ Torrs. For example, at a pressure of 1 Torr, $M(1) = 0.5$, which means half of the myoglobin (i.e. 50%) is oxygen saturated. (Note: More precisely, you need to use something called the "partial pressure", but the distinction is not important for this problem.) Likewise, the function calculates the fraction of hemoglobin saturated with oxygen at a given pressure $p$. [UW]

a. The graphs of M(P) and H(P) are given here on the domain $0 \le p \le 100$




Which is which?

b. If the pressure in the lungs is 100 Torrs, what is the level of oxygen saturation of the hemoglobin in the lungs?

c. The pressure in an active muscle is 20 Torrs. What is the level of oxygen saturation of myoglobin in an active muscle? What is the level of hemoglobin in an active muscle?

d. Define the efficiency of oxygen transport at a given pressure $p$ to be M(p) – H(p) . What is the oxygen transport efficiency at 20 Torrs? At 40 Torrs? At 60 Torrs? Sketch the graph of M(p) – H(p) ; are there conditions under which transport efficiency is maximized (explain)?

**Technology (NJSLS Career Readiness, Life Literacies, and Key Skills)**

- 9.4.12.TL.1: Assess digital tools based on features such as accessibility options, capacities, and utility for accomplishing a specific task (e.g., W.11-12.6.).
  - ⇒ Example: Students will use digital tools such as TI-84, Desmos, Google Suite, etc. to access, manage, evaluate, and synthesize information in order to solve problems individually and collaboratively to

create and communicate knowledge.

**21st Century Skills (NJSLS Career Readiness, Life Literacies, and Key Skills)**

- 9.4.12.CT.1: Identify problem-solving strategies used in the development of an innovative product or practice (e.g., 1.1.12acc.C1b, 2.2.12.PF.3).
  - ⇒ Example: Career-ready individuals readily recognize problems in the workplace, understand the nature of the problem, and devise effective plans to solve the problem. They are aware of problems when they occur and take action quickly to address the problem; they thoughtfully investigate the root cause of the problem prior to introducing solutions. They carefully consider the options to solve the problem. Once a solution is agreed upon, they follow through to ensure the problem is solved, whether through their own actions or the actions of others.
  - ⇒ Example: Throughout this unit, students work to improve their critical-thinking skills and problem-solving techniques.

**NJ SEL Competencies**

- Self-Awareness: Recognize one’s feelings and thoughts
- Self-Management: Understand and practice strategies for managing one’s own emotions, thoughts, and behaviors

**Climate Change**

- ECOSYSTEM Students will apply rational equations to analyze the decline of the deer population in the Kailbab Pleateau in Arizona and use models to make predictions. (Glencoe Advanced Math Concepts 2006 Textbook p. 242 #37)

**Ecology** In the early 1900s, the deer population of the Kaibab Plateau in Arizona experienced a rapid increase because hunters had reduced the number of natural predators. The food supply was not great enough to support the increased population, eventually causing the population to decline. The deer population for the period 1905 to 1930 can be modeled by $f(x) = -0.125x^5 + 3.125x^4 + 4000$, where $x$ is the number of years from 1905.

**a.** Graph the function.

**b.** Use the model to determine the population in 1905.

**c.** Use the model to determine the population in 1920.

**d.** According to this model, when did the deer population become zero?

## Culturally Relevant Connections

- Integrate Relevant Word Problems: Contextualize equations using word problems that reference student interests and cultures. Example: When learning to interpret functions, problems that relate to student interests such as music, sports and art enable the students to understand and relate to the concept in a more meaningful way.
- Encourage Student Leadership: Create an avenue for students to propose problem solving strategies and potential projects. Example: Students can learn to construct and compare non-linear functions by creating problems together and deciding if the problems fit the necessary criteria. This experience will allow students to discuss and explore their current level of understanding by applying the concepts to relevant real-life experiences.
- Present New Concepts Using Student Vocabulary: Use student diction to capture attention and build understanding before using academic terms. Example: Teach math vocabulary in various modalities for students to remember. Use multi-modal activities, analogies, realia, visual cues, graphic representations, gestures, pictures and cognates. Directly explain and model the idea of vocabulary words having multiple meanings. Students can create the Word Wall with their definitions and examples to foster ownership.

## Accommodations

| **Special Education/ 504/ At Risk Students Accommodations & Modifications:** | **ELL:** |
| --- | --- |
| | ○ Translation dictionary |

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| ○ Anchor charts to model strategies<br>○ Review Algebra concepts to ensure students have the information needed to progress in understanding<br>○ Provide reference sheets that list formulas, step by step procedures, theorems, and modeling of strategies<br>○ Teacher modeling of thinking processes<br>○ Introduce concepts embedded in real-life to help students relate to the mathematics involved<br>○ Graphing calculator to assist with computations and graphing of trigonometric functions<br>○ Utilize technology through interactive sites to represent nonlinear data | ○ Sentence stems<br>○ Pre-teach pertinent vocabulary<br>○ Word wall with visual representations of mathematical terms<br>○ Graphic organizers to help students interpret the meaning in an expression or equation in context |

**Enrichment**

- Extended learning goals:
  - ⇒ Students will extend their knowledge of polynomial functions to graphic addition, in which a polynomial function can be written as the sum of two other functions that are easier to graph.
  - ⇒ Students will extend their knowledge of locating zeros of a polynomial function to applying the bisection method to approximate zeros of a polynomial function.

| Unit 2: Trigonometric Functions | |
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| **Desired Outcomes** | |
| **Established Goals: NJSLS**<br><br>○ Define trigonometric ratios and solve problems involving right triangles (G.SRT.6, G.SRT.7, G.SRT.8).<br>○ Apply trigonometry to general triangles (G.SRT.10, G.SRT.11).<br>○ Extend the domain of trigonometric functions using the unit circle (F.TF.3, F.TF.4).<br>○ Model periodic phenomena with trigonometric functions (F.TF.6, F.TF.7).<br><br>**NJSLS Mathematical Practices**<br>1. Make sense of problems and persevere in solving them.<br>2. Reason abstractly and quantitatively.<br>3. Construct viable arguments and critique the reasoning of others.<br>4. Model with mathematics.<br>5. Use appropriate tools strategically.<br>6. Attend to precision.<br>7. Look for and make use of structure.<br>8. Look for and express regularity in repeated reasoning. | |
| **Enduring Understandings:**<br><br>○ Given 3 parts of any type of triangle you can find side lengths and angle measurements<br>○ There are multiple ways to measure the various aspects of the Unit Circle<br>○ Periodic phenomena can be represented by trigonometric functions<br>○ Different trigonometric function have different characteristics | **Essential Questions:**<br><br>○ Why is it necessary to have multiple ways to solve triangles?<br>○ How can periodic phenomena be represented using trigonometric functions?<br>○ What are the characteristics of trigonometric functions?<br>○ What role does critical vocabulary play in understanding trigonometric functions? |

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| | ○ What are radians and how can we use them to describe the measurement of an angle?<br>○ How can you use right triangles and the unit circle to find the trigonometric ratios of special angles?<br>○ How do the transformations and characteristics of trigonometric functions affect their graphs? |
| **<u>Students will know:</u>**<br><br>○ The relationship between arc length and the measure of the angle in radians trigonometric values are ratios, that are constant regardless of the size of the triangle.<br>○ The period of a trigonometric function can represent different real life situationThe general shape of all six trigonometric functions.<br>○ The correct terms and usage for all six trigonometric functions.<br>○ Key vocabulary: trigonometric functions, sine, cosine, tangent, cosecant, secant, cotangent, reciprocal function, inverse sine, inverse cosine, inverse tangent, radian, coterminal angles, reference angle, unit circle, circular function, period, sinusoid, amplitude, frequency, phase shift, Law of Sines, Law of Cosines | **<u>Students will be able to:</u>**<br><br>Trigonometric Functions<br>○ Convert decimal degree measures to degrees, minutes, and seconds and vice versa.<br>○ Find the number of degrees in a given number of rotations.<br>○ Identify angles that are coterminal with a given angle.<br>○ Find the values of trigonometric ratios for acute angles of right triangles.<br>○ Find the values of six trigonometric functions using the unit circle.<br>○ Find the values of six trigonometric functions of an angle in standard position given a point on its terminal side.<br>○ Use trigonometry to find the measures of the sides of right triangles.<br>○ Evaluate inverse trigonometric functions.<br>○ Find missing angle measurements<br>○ Solve right triangles<br>○ Solve triangles by using the Law of Sines if the measures of two angles and a side are given.<br>○ Find the area of a triangle if the measures of two sides and the included angle or the measures of two angles and a side are given. |

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| | ○ Determine whether a triangle has zero, one, or two solutions.<br>○ Solve triangles using the Law of Sines.<br>○ Solve triangles using the Law of Cosines.<br>○ Find the area of triangles if the measure of the three sides are given.<br>Graphs of Trigonometric Functions<br>○ Changes from radian measure to degree measure, and vice versa.<br>○ Find the length of an arc given the measure of the central angle.<br>○ Find the area of a sector.<br>○ Find linear and angular velocity.<br>○ Use the graphs of the sine and cosine functions.<br>○ Find the amplitude and period for sine and cosine functions.<br>○ Write equations of sine and cosine functions given the amplitude and period.<br>○ Find the phase shift and the vertical translation for sine and cosine functions.<br>○ Write the equations of sine and cosine functions given the amplitude, period, phase shift, and vertical translation.<br>○ Sketch compound functions (example: y=x+sinx).<br>○ Create a sine or cosine function that models real-world data.<br>○ Use sinusoidal functions to solve problems.<br>○ Graph and write equations for tangent, cotangent, secant, and cosecant functions. |
| **Assessment Evidence** | |
| **<u>Suggested Performance Tasks:</u>** | **<u>Required District/State Assessments:</u>** |

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| ○ Sinusoidal Functions Project: You have been asked by a major publishing company to write a book that connects graphs of trigonometric functions to sound or circular movement applications. Your publicist wants you to create an advertisement for your book's upcoming date. Create a professional and intriguing advertisement that will please your publicist and generate interest for your book by showcasing the trigonometric graphing connection to your topic.<br>○ Math Assessment Project: [Representing Trigonometric Functions]() | ○ Unit Assessment<br>○ SGO Assessments<br><br>**Suggested Formative/Summative Assessments:**<br><br>○ Describe Learning Vertically<br>○ Identify Key Building Blocks<br>○ Make Connections (between and among key building blocks)<br>○ Short/Extended Constructed Response Items<br>○ Multiple-Choice Items (where multiple answer choices may be correct)<br>○ Drag and Drop Items<br>○ Use of Equation Editor<br>○ Quizzes<br>○ Journal Entries/Reflections/Quick-Writes<br>○ Accountable talk<br>○ Projects<br>○ Portfolio<br>○ Observation<br>○ Graphic Organizers/ Concept Mapping<br>○ Presentations<br>○ Role Playing<br>○ Teacher-Student and Student-Student Conferencing<br>○ Homework |

## Learning Plan

**Learning Activities:**

- Starter exercises
- Guided notes as necessary
- In class activities (QR codes, scavenger hunt, interactive exercises, exit ticket, etc.)

## Related Standards

**Interdisciplinary connections**

- Science Connection (HS-PS4-1)
  - ⇒ Example page 363 TE Have students research and possibly observe what the graphs of the sound patterns of a person's live voice, a song recorded by Frank Sinatra, a rock concert, and a healthy and an unhealthy heart would look like. Are any of these patterns periodic functions?
  - ⇒ Example: Glencoe Advanced Math Concepts Textbook Page 375 #58.
- Language Arts Connection (NJSLS.W.9-10.1.C)
  - ⇒ Example: ongoing throughout unit
- Construction (9.3.12.AC-CST.7)
  - ⇒ Example: Glencoe Advanced Math Concepts Textbook:
    - Page 290 #31

      **Architecture** The angle of inclination of the sun affects the heating and cooling of buildings. The angle is greater in the summer than the winter. The sun's angle of inclination also varies according to the latitude. The sun's angle of inclination at noon equals $90° - L - 23.5° \times \cos\left[\frac{(N + 10)360}{365}\right]$. In this expression, $L$ is the latitude of the building site, and $N$ is the number of days elapsed in the year.

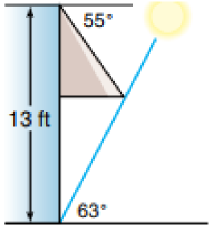
      **a.** The latitude of Brownsville, Texas, is 26°. Find the angle of inclination for Brownsville on the first day of summer (day 172) and on the first day of winter (day 355).

      **b.** The latitude of Nome, Alaska, is 64°. Find the angle of inclination for Nome on the first day of summer and on the first day of winter.

      **c.** Which city has the greater change in the angle of inclination?
    - Page 317 #30, 34

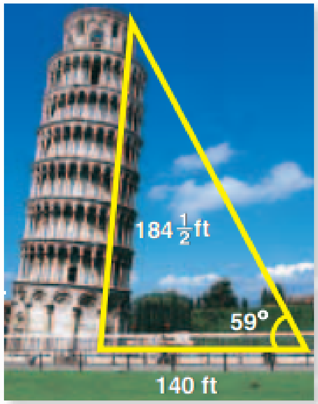
**Architecture** The center of the Pentagon in Arlington, Virginia, is a courtyard in the shape of a regular pentagon. The pentagon could be inscribed in a circle with radius of 300 feet. Find the area of the courtyard.


**Architecture** An architect is designing an overhang above a sliding glass door. During the heat of the summer, the architect wants the overhang to prevent the rays of the sun from striking the glass at noon. The overhang has an angle of depression of 55° and starts 13 feet above the ground. If the angle of elevation of the sun during this time is 63°, how long should the architect make the overhang?




- Page 325 #35

**Architecture** The original height of the Leaning Tower of Pisa was $184\frac{1}{2}$ feet. At a distance of 140 feet from the base of the tower, the angle of elevation from the ground to the top of the tower is 59°. How far is the tower leaning from the original vertical position?




**<u>Technology (NJSLS Career Readiness, Life Literacies, and Key Skills)</u>**

- 9.4.12.TL.2: Generate data using formula-based calculations in a spreadsheet and draw conclusions about the data.
  - ⇒ Example: Students will explore trigonometry formulas and graphing trigonometric functions in Google Spreadsheets.

**21st Century Skills (NJSLS Career Readiness, Life Literacies, and Key Skills)**

- 9.4.12.IML.3: Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions (e.g., S-ID.B.6a., 8.1.12.DA.5, 7.1.IH.IPRET.8)
  - ⇒ Example: Students use graphing calculators to calculate the reference angle or the sides of a right triangle associated with trigonometric functions.

**NJ SEL Competencies**

- Social Awareness: Recognize and identify the thoughts, feelings, and perspectives of others
- Responsible Decision-Making: Develop, implement and model effective problem solving and critical thinking skills

**Climate Change**

- ENERGY (HS-ESS2-4). Use a model to describe how variations in the flow of energy into and out of Earth's systems result in changes in climate.
  - ⇒ Example: Glencoe Advanced Math Concepts 2006 Textbook
    - Page 391 #6

**Meteorology** The average monthly temperatures for the city of Seattle, Washington, are given below.

| Jan. | Feb. | March | April | May | June | July | Aug. | Sept. | Oct. | Nov. | Dec. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 41° | 44° | 47° | 50° | 56° | 61° | 65° | 66° | 61° | 54° | 46° | 42° |

a. Find the amplitude of a sinusoidal function that models the monthly temperatures.

b. Find the vertical shift of a sinusoidal function that models the monthly temperatures.

c. What is the period of a sinusoidal function that models the monthly temperatures?

d. Write a sinusoidal function that models the monthly temperatures, using $t = 1$ to represent January.

e. According to your model, what is the average monthly temperature in February? How does this compare to the actual average?

f. According to your model, what is the average monthly temperature in October? How does this compare to the actual average?

- Page 392 #12

  **Meteorology** The mean average temperature in Buffalo, New York, is 47.5°. The temperature fluctuates 23.5° above and below the mean temperature. If $t = 1$ represents January, the phase shift of the sine function is 4.

  a. Write a model for the average monthly temperature in Buffalo.

  b. According to your model, what is the average temperature in March?

  c. According to your model, what is the average temperature in August?

- Page 393 #13

**Meteorology** The average monthly temperatures for the city of Honolulu, Hawaii, are given below.

| Jan. | Feb. | March | April | May | June | July | Aug. | Sept. | Oct. | Nov. | Dec. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 73° | 73° | 74° | 76° | 78° | 79° | 81° | 81° | 81° | 80° | 77° | 74° |

a. Find the amplitude of a sinusoidal function that models the monthly temperatures.

b. Find the vertical shift of a sinusoidal function that models the monthly temperatures.

c. What is the period of a sinusoidal function that models the monthly temperatures?

d. Write a sinusoidal function that models the monthly temperatures, using $t = 1$ to represent January.

e. According to your model, what is the average temperature in August? How does this compare to the actual average?

f. According to your model, what is the average temperature in May? How does this compare to the actual average?

- Page 393 #16

**Meteorology** The table at the right contains the times that the sun rises and sets in the middle of each month in New York City, New York. Suppose the number 1 represents the middle of January, the number 2 represents the middle of February, and so on.

a. Find the amount of daylight hours for the middle of each month.

b. What is the amplitude of a sinusoidal function that models the daylight hours?

c. What is the vertical shift of a sinusoidal function that models the daylight hours?

d. What is the period of a sinusoidal function that models the daylight hours?

e. Write a sinusoidal function that models the daylight hours.

| Month | Sunrise A.M. | Sunset P.M. |
|---|---|---|
| January | 7:19 | 4:47 |
| February | 6:56 | 5:24 |
| March | 6:16 | 5:57 |
| April | 5.25 | 6:29 |
| May | 4:44 | 7:01 |
| June | 4:24 | 7:26 |
| July | 4:33 | 7:28 |
| August | 5:01 | 7:01 |
| September | 5:31 | 6:14 |
| October | 6:01 | 5:24 |
| November | 6:36 | 4:43 |
| December | 7:08 | 4:28 |

## Culturally Relevant Connections

- *Integrate Relevant Word Problems:* Contextualize equations using word problems that reference student interests and cultures. Example: When interpreting functions, problems that relate to student interests such as music, sports and art enable the students to understand and relate to the concept in a more meaningful way.
- *Encourage Student Leadership:* Create an avenue for students to propose problem solving strategies and potential projects. Example: Students can learn to construct and compare trigonometric functions by creating problems together and deciding if the problems fit the necessary criteria. This experience will allow students to discuss and explore their current level of understanding by applying the concepts to relevant real-life experiences.
- *Present New Concepts Using Student Vocabulary:* Use student diction to capture attention and build understanding before using academic terms. Example: Teach math vocabulary in various modalities for

students to remember. Use multi-modal activities, analogies, realia, visual cues, graphic representations, gestures, pictures and cognates. Directly explain and model the idea of vocabulary words having multiple meanings. Students can create the Word Wall with their definitions and examples to foster ownership.

- *Everyone has a Voice:* Create a classroom environment where students know that their contributions are expected and valued. Example: Norms for sharing are established that communicate a growth mindset for mathematics. All students are capable of expressing mathematical thinking and contributing to the classroom community. Students learn new ways of looking at problem solving by working with and listening to each other.

## Accommodations

**<u>Special Education/ 504/ At Risk Students Accommodations & Modifications:</u>**

- Anchor charts to model strategies
- Review Algebra concepts to ensure students have the information needed to progress in understanding
- Provide reference sheets that list formulas, step by step procedures, theorems, and modeling of strategies
- Teacher modeling of thinking processes
- Introduce concepts embedded in real-life to help students relate to the mathematics involved
- Graphing calculator to assist with computations and graphing of trigonometric functions
- Utilize technology through interactive sites to represent nonlinear data
- Use videos to reinforce skills and thinking behind concepts.
- Highlight and label solution steps for multi-step problems in different colors.
- Encourage students to verbalize their thinking

**<u>ELL:</u>**

- Translation dictionary
- Sentence stems
- Pre-teach pertinent vocabulary
- Word wall with visual representations of mathematical terms
- Graphic organizers to help students interpret the meaning in an expression or equation in context
- Use anchor charts with important terms, problem solving approaches, pictures and translations as needed.
- Modify the linguistic complexity of tasks by rephrasing math problems.
- Teacher models the thinking process used and the academic vocabulary needed to solve multistep problems that require students to interpret units consistently and accurately.
- The students can explain through a “think aloud” and demonstrate how they solved the problem or explain their work.
- Have students work in triads or small groups

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| while working in small groups by asking, assessing, and advancing questions.<br>○ Extend the domain of trigonometric functions using the unit circle in order to relate the idea of degrees to radians.<br>○ Explain the background of trigonometric functions and connect to real life by explaining how it is used.<br>○ Allow time for questions to check on student understanding.<br>○ Provide students with a graphic organizer for the Unit Circle.<br>○ Review Geometry concepts of proving triangles congruent when determining area of triangles using sine.<br>○ Have students use calculators to find values of the sine and cosine functions to help graph. | where they are able to support each other's learning by giving each other input and filling in gaps in background. |

## Enrichment

- Extended learning goals:
  - ⇒ Students can apply right triangles on a coordinate plane to determine the area of obtuse or acute triangles (5.2 Enrichment: Using Right Triangles to Find the Area of Another Triangle.
  - ⇒ Students can apply trigonometric functions to determine the area of polygons that are inscribed and circumscribed (5.3 Enrichment: Areas of Polygons and Circles).
  - ⇒ Students can build and use a hypsometer to determine the height of tall objects using trigonometric functions (5.4 Enrichment: Making and Using a Hypsometer).
  - ⇒ Students can apply trigonometric functions and error analysis to show that an angle trisection method is not valid (5.5 Enrichment: Disproving Angle Trisection).
  - ⇒ Students can apply trigonometry to solving spherical triangles (5.7 Enrichment: Spherical Triangles).
  - ⇒ Students can use trigonometry and the Pythagorean Theorem to solve for the third side of any triangle (5.8 Enrichment: The Law of Cosines and the Pythagorean Theorem).
  - ⇒ Students can calculate the angular acceleration of an object in motion (6.2 Enrichment: Angular

Acceleration).

⇒ Students can explore, describe, and make visual representations of real-world periodic phenomena (6.3 Enrichment: Periodic Phenomena).

⇒ Students can apply trigonometric functions to determine the motion of a floating object and its mass (6.4 Enrichment: Mass of a Floating Object).

⇒ Students can describe and graph translations of trigonometric functions (6.5 Enrichment: Translating Graphs of Trigonometric Functions).

# Unit 3: Analytic Trigonometry

## Desired Outcomes

**Established Goals: NJSLS**

- Prove and apply trigonometric identities (F.TF.8, F.TF.9).

**NJSLS Mathematical Practices**

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

| Enduring Understandings: | Essential Questions: |
| --- | --- |
| ○ Trigonometric identities can be used to identify non special angles<br>○ There is more than one way to prove a trig identity | ○ What is an identity?<br>○ How can trigonometric identities be used to solve trigonometric equations?<br>○ Why is there more than one way to prove a trig identity? |
| **Students will know:** | **Students will be able to:** |
| ○ The sum, difference, half angle and double angle formulas<br>○ The Pythagorean, reciprocal, symmetry, and | ○ Identify and use reciprocal identities, quotient identities, Pythagorean identities, symmetry identities, and opposite-angle identities. |

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| quotient identities<br>○ The formulas for the Law of Sines and Law of Cosines<br>○ Key vocabulary: trigonometric identity, reciprocal identity, quotient identity, Pythagorean identity, cofunction, verify an identity, sum identity, double angle identity, half-angle identity | ○ Use the basic trigonometric identities to verify other identities.<br>○ Find numerical values of trigonometric functions.<br>○ Use the sum and difference identities for the sine, cosine, and tangent functions.<br>○ Use the double- and half-angle identities for the sine, cosine, and tangent functions. |
| **Assessment Evidence** | |
| **Suggested Performance Tasks:**<br><br>○ Proving Trigonometric Identities: Complete the Proof<br>○ Electricity A current in a wire in a magnetic field causes a force to act on the wire. The strength of the magnetic field can be determined using the formula B=(F cscθ)/Il, where F is the force on the wire, I is the current in the wire, l is the length of the wire, and θ is the angle the wire makes with the magnetic field. Some physics books give the formula F=IlB sinθ. Show that the two formulas are equivalent.<br>○ In this problem, you will investigate methods used to solve trigonometric equations. Consider 1=2 sin x.<br>a. Numerical: Isolate the trigonometric function in the equation so that sin x is the only expression on one side of the equation.<br>b. Graphical: Graph the left and right sides of the equation you found in part a on the same graph over [0,2π). Locate any points of intersection and express the values in | **Required District/State Assessments:**<br><br>○ Unit Assessment<br>○ SGO Assessments<br><br>**Suggested Formative/Summative Assessments:**<br><br>○ Describe Learning Vertically<br>○ Identify Key Building Blocks<br>○ Make Connections (between and among key building blocks)<br>○ Short/Extended Constructed Response Items<br>○ Multiple-Choice Items (where multiple answer choices may be correct)<br>○ Drag and Drop Items<br>○ Use of Equation Editor<br>○ Quizzes<br>○ Journal Entries/Reflections/Quick-Writes<br>○ Accountable talk<br>○ Projects<br>○ Portfolio<br>○ Observation<br>○ Graphic Organizers/ Concept Mapping |

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| terms of radians.<br>c. Geometric: Use the unit circle to verify the answers you found in part b.<br>d. Graphical: Graph the left and right sides of the equation you found in part a on the same graph over $-2\pi<x<2\pi$. Locate any points of intersection and express the values in terms of radians.<br>e. Verbal: Make a conjecture as to the solutions of $1=2 \sin x$. Explain your reasoning. | ○ Presentations<br>○ Role Playing<br>○ Teacher-Student and Student-Student Conferencing<br>○ Homework |

## Learning Plan

**<u>Learning Activities:</u>**

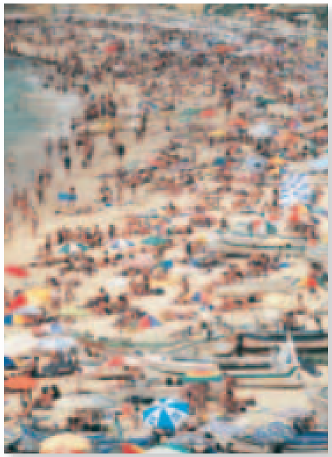
- Starter exercises
- Guided notes as necessary
- In class activities (QR codes, scavenger hunt, interactive exercises, exit ticket, etc.)

## Related Standards

**<u>Interdisciplinary connections</u>**

- Language Arts Connection (NJSLS.W.9-10.1.C)
  - ⇒ Example: ongoing throughout unit
- Career and Technical Education
  - ⇒ 9.3.HL-BRD.5 Determine processes for product design and production and how that work contributes to an understanding of the biotechnology product development process.
    - ■ Example: Glencoe Advanced Math Concepts Textbook Page 429 #56.

**Dermatology** It has been shown that skin cancer is related to sun exposure. The rate $W$ at which a person's skin absorbs energy from the sun depends on the energy $S$, in watts per square meter, provided by the sun, the surface area $A$ exposed to the sun, the ability of the body to absorb energy, and the angle $\theta$ between the sun's rays and a line perpendicular to the body. The ability of an object to absorb energy is related to a factor called the *emissivity*, $e$, of the object. The emissivity can be calculated using the formula $e = \frac{W \sec \theta}{AS}$.



a. Solve this equation for $W$. Write your answer using only $\sin \theta$ or $\cos \theta$.

b. Find $W$ if $e = 0.80$, $\theta = 40°$, $A = 0.75 \text{ m}^2$, and $S = 1000 \text{ W/m}^2$.

## Technology (NJSLS Career Readiness, Life Literacies, and Key Skills)

- 9.4.12.TL.1: Assess digital tools based on features such as accessibility options, capacities, and utility for accomplishing a specific task (e.g., W.11-12.6.).
  - ⇒ Example: Students will use digital tools such as TI-84, Desmos, Google Suite, etc. to access, manage, evaluate, and synthesize information in order to solve problems individually and collaboratively to create and communicate knowledge.

## 21st Century Skills (NJSLS Career Readiness, Life Literacies, and Key Skills)

- 9.4.12.IML.3: Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions (e.g., S-ID.B.6a., 8.1.12.DA.5, 7.1.IH.IPRET.8)
  - ⇒ Example: Students use graphing calculators to calculate numerical values of trigonometric functions.

## NJ SEL Competencies

- Social Awareness: Demonstrate an understanding of the need for mutual respect when viewpoints differ

- Responsible Decision-Making: Develop, implement and model effective problem solving and critical thinking skills

## Culturally Relevant Connections

- *Integrate Relevant Word Problems:* Contextualize equations using word problems that reference student interests and cultures. Example: When interpreting functions, problems that relate to student interests such as music, sports and art enable the students to understand and relate to the concept in a more meaningful way.
- *Encourage Student Leadership:* Create an avenue for students to propose problem solving strategies and potential projects. Example: Students can learn to construct and compare different methods of simplification of trigonometric identities by creating solutions together and deciding the pros and cons of each method. This experience will allow students to discuss and explore their current level of understanding by applying the concepts to relevant real-life experiences.
- *Present New Concepts Using Student Vocabulary:* Use student diction to capture attention and build understanding before using academic terms. Example: Teach math vocabulary in various modalities for students to remember. Use multi-modal activities, analogies, realia, visual cues, graphic representations, gestures, pictures and cognates. Directly explain and model the idea of vocabulary words having multiple meanings. Students can create the Word Wall with their definitions and examples to foster ownership.
- *Everyone has a Voice*: Create a classroom environment where students know that their contributions are expected and valued. Example: Norms for sharing are established that communicate a growth mindset for mathematics. All students are capable of expressing mathematical thinking and contributing to the classroom community. Students learn new ways of looking at problem solving by working with and listening to each other.

## Accommodations

| **Special Education/ 504/ At Risk Students Accommodations & Modifications:** | **ELL:** |
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| ○ Anchor charts to model strategies<br>○ Review Algebra concepts to ensure students have the information needed to progress in | ○ Translation dictionary<br>○ Sentence stems<br>○ Pre-teach pertinent vocabulary<br>○ Word wall with visual representations of |

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| understanding<br>○ Provide reference sheets that list formulas, step by step procedures, theorems, and modeling of strategies<br>○ Teacher modeling of thinking processes<br>○ Introduce concepts embedded in real-life to help students relate to the mathematics involved<br>○ Use calculators to confirm trigonometric identities in using degrees to have concrete examples.<br>○ Utilize technology through interactive sites to represent nonlinear data<br>○ Use videos to reinforce skills and thinking behind concepts.<br>○ Highlight and label solution steps for multi-step problems in different colors.<br>○ Encourage students to verbalize their thinking while working in small groups by asking, assessing, and advancing questions.<br>○ Allow time for questions to check on student understanding.<br>○ Allow students to keep a list of trigonometric identities they learn throughout the chapter as a reference.<br>○ Compare and contrast simple algebraic equations or right triangles to trigonometric identities by using counterexamples.<br>○ Understand how to interpret $\pm$ and $\mp$.<br>○ Compare solving trigonometric equations to factor by grouping in Algebra. | mathematical terms<br>○ Graphic organizers to help students interpret the meaning in an expression or equation in context<br>○ Use anchor charts with important terms, problem solving approaches, pictures and translations as needed.<br>○ Modify the linguistic complexity of tasks by rephrasing math problems.<br>○ Teacher models the thinking process used and the academic vocabulary needed to solve multistep problems that require students to interpret units consistently and accurately.<br>○ The students can explain through a “think aloud” and demonstrate how they solved the problem or explain their work.<br>○ Have students work in triads or small groups where they are able to support each other’s learning by giving each other input and filling in gaps in background. |
| **Enrichment** | |

- Extended learning goals:
  - ⇒ Students can calculate the maximum height of a projectile using trigonometric functions (7.1 Enrichment: The Physics of Soccer).
  - ⇒ Students can create and manipulate their own trigonometric identities (7.2 Enrichment: Building from 1 = 1).
  - ⇒ Students can apply trigonometric identities to simplify trigonometric products (7.3 Enrichment: The Physics of Soccer).

# Unit 4: Vectors and Parametric Equations

## Desired Outcomes

**Established Goals:  NJSLS**

- Represent and model with vector quantities (N.VM.1, N.NM.2, N.VM.3).
- Perform operations on vectors (N.VM.4a, N.VM.4b, N.VM.4c, N.VM.5a, N.VM.5b).

**NJSLS Mathematical Practices**

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

| Enduring Understandings: | Essential Questions: |
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| ○ Vectors can be used to model real world applications involving quantity and direction. | ○ How can you represent physical quantities that you cannot see?<br>○ How can problems with 3 variables be represented in 2 dimensions?<br>○ How can I solve problems involving forces mathematically? |

| Students will know: | Students will be able to: |
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| ○ Basic operations of vectors. | ○ Find equal, opposite, and parallel vectors. |

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| ○ Vector applications to solve naturally occurring problems.<br>○ Key vocabulary: vector, initial point, terminal point, standard position, direction, magnitude, quadrant bearing, true bearing, parallel vectors, equivalent vectors, opposite vectors, resultant, zero vector, component form, unit vector, dot product, orthogonal, z-axis, ordered triple, cross product, parametric equation | ○ Add and subtract vectors geometrically.<br>○ Find ordered pairs that represent vectors.<br>○ Add, subtract, multiply, and find the magnitude of vectors algebraically.<br>○ Add, subtract, and find the magnitude of vectors in three dimensional space.<br>○ Find the inner and cross products of two vectors.<br>○ Determine whether two vectors are perpendicular.<br>○ Solve problems using vectors and right triangle trigonometry.<br>○ Write vector and parametric equations of lines.<br>○ Graph parametric equations.<br>○ Model the motion of a projectile using parametric equations.<br>○ Solve problems related to the motion of a projectile, its trajectory, and range. |
| **Assessment Evidence** | |
| **<u>Suggested Performance Tasks:</u>**<br><br>○ Modeling Motion Project: Students will apply knowledge from unit to solve a variety of real-world, physics application problems.<br>○ Students use what they have learned about vectors in two and three dimensions to captain a riverboat.<br>    a. Have each student determine a speed that the boat will travel perpendicular to the shore. Have students calculate the speeds at which their boats are crossing the river and the angle at which their boats are traveling with respect to the shore when | **<u>Required District/State Assessments:</u>**<br><br>○ Unit Assessment<br>○ SGO Assessments<br><br>**<u>Suggested Formative/Summative Assessments:</u>**<br><br>○ Describe Learning Vertically<br>○ Identify Key Building Blocks<br>○ Make Connections (between and among key building blocks)<br>○ Short/Extended Constructed Response Items<br>○ Multiple-Choice Items (where multiple answer choices may be correct) |

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| the current is 4 mph heading downstream.<br>b. For a second trip, students design boat ramps, labeling the length and angle to the horizontal. Have each student determine how much work is done when a force of 520 N is used to push a barrel up the ramp.<br>c. Have two students work together to use their boats to pull a disabled boat. They should attach their towlines—one west of north, the other east of north—the same number of degrees. If each boat exerts a constant force of 2.25 × 106 N depressed 15° below the point where the lines attach to the disabled boat, have students determine a three-dimensional vector to describe the force from each boat. | ○ Drag and Drop Items<br>○ Use of Equation Editor<br>○ Quizzes<br>○ Journal Entries/Reflections/Quick-Writes<br>○ Accountable talk<br>○ Projects<br>○ Portfolio<br>○ Observation<br>○ Graphic Organizers/ Concept Mapping<br>○ Presentations<br>○ Role Playing<br>○ Teacher-Student and Student-Student Conferencing<br>○ Homework |

## Learning Plan

**Learning Activities:**

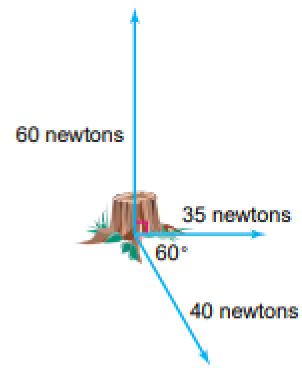
- Starter exercises
- Guided notes as necessary
- In class activities (QR codes, scavenger hunt, interactive exercises, exit ticket, etc.)

## Related Standards

**Interdisciplinary connections**

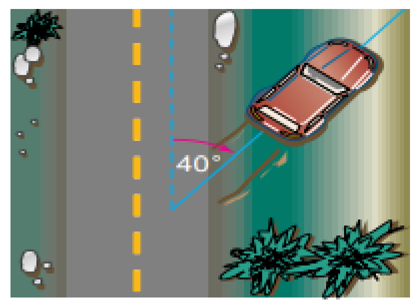
- Language Arts Connection (NJSLS.W.9-10.1.C)
  - ⇒ Example: ongoing throughout unit
- Science Connection (HS-PS2-1)
  - ⇒ Example: Example: Glencoe Advanced Math Concepts Textbook Page 491 #38.

**Physics** Three workers are pulling on ropes attached to a tree stump as shown in the diagram. Find the magnitude and direction of the resultant force on the tree. *A newton (N) is a unit of force used in physics. A force of one newton will accelerate a one-kilogram mass at a rate of one meter per second squared.*




- Career and Technical Education (9.3.LW-EFM.5)
  - ⇒ Example: Glencoe Advanced Math Concepts Textbook Page 492 #41.

**Police Investigation** Police officer Patricia Malloy is investigating an automobile accident. A car slid off an icy road at a 40° angle with the center line of the road with an initial velocity of 47 miles per hour. Use the drawing to determine the initial horizontal and vertical components of the car's velocity.




**<u>Technology (NJSLS Career Readiness, Life Literacies, and Key Skills)</u>**

- 9.4.12.TL.2: Generate data using formula-based calculations in a spreadsheet and draw conclusions about the data.
  - ⇒ Students will utilize Google Sheets to develop and test a formula to calculate the dot product and cross product. Students test their formula for accuracy and compare the efficiency and accuracy with hand calculations.

**21st Century Skills (NJSLS Career Readiness, Life Literacies, and Key Skills)**

- 9.4.12.IML.3: Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions (e.g., S-ID.B.6a., 8.1.12.DA.5, 7.1.IH.IPRET.8)
  - ⇒ Example: Students use graphing calculators to model parametric equations (Glencoe Advanced Math Concepts Textbook Page 526 - Lab)

**NJ SEL Competencies**

- Relationship Skills: Establish and maintain healthy relationships
- Self-Awareness: Recognize one's personal traits, strengths and limitations

## Culturally Relevant Connections

- *Integrate Relevant Word Problems:* Contextualize equations using word problems that reference student interests and cultures. Example: When interpreting functions, problems that relate to student interests such as music, sports and art enable the students to understand and relate to the concept in a more meaningful way.
- *Encourage Student Leadership:* Create an avenue for students to propose problem solving strategies and potential projects. Example: Students can learn to construct and compare vectors in the 3-D space by creating problems together and deciding if the problems fit the necessary criteria. This experience will allow students to discuss and explore their current level of understanding by applying the concepts to relevant real-life experiences.
- *Present New Concepts Using Student Vocabulary:* Use student diction to capture attention and build understanding before using academic terms. Example: Teach math vocabulary in various modalities for students to remember. Use multi-modal activities, analogies, realia, visual cues, graphic representations, gestures, pictures and cognates. Directly explain and model the idea of vocabulary words having multiple meanings. Students can create the Word Wall with their definitions and examples to foster ownership.
- *Everyone has a Voice:* Create a classroom environment where students know that their contributions are expected and valued. Example: Norms for sharing are established that communicate a growth mindset for mathematics. All students are capable of expressing mathematical thinking and contributing to the classroom community. Students learn new ways of looking at problem solving by working with and listening to each other.

| Accommodations | |
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| **<u>Special Education/ 504/ At Risk Students Accommodations & Modifications:</u>**<br><br>○ Anchor charts to model strategies<br>○ Review Algebra concepts to ensure students have the information needed to progress in understanding<br>○ Provide reference sheets that list formulas, step by step procedures, theorems, and modeling of strategies<br>○ Teacher modeling of thinking processes<br>○ Introduce concepts embedded in real-life to help students relate to the mathematics involved<br>○ Graphing calculator to assist with computations and graphing of.<br>○ Utilize technology through interactive sites to represent vectors.<br>○ Use videos to reinforce skills and thinking behind concepts.<br>○ Highlight and label solution steps for multi-step problems in different colors.<br>○ Encourage students to verbalize their thinking while working in small groups by asking, assessing, and advancing questions.<br>○ Allow time for questions to check on student understanding.<br>○ Use a graphing calculator to obtain the components of the cross product of two vectors in space.<br>○ Encourage students to include a drawing with | **<u>ELL:</u>**<br><br>○ Translation dictionary<br>○ Sentence stems<br>○ Pre-teach pertinent vocabulary<br>○ Word wall with visual representations of mathematical terms<br>○ Graphic organizers to help students interpret the meaning in an expression or equation in context<br>○ Use anchor charts with important terms, problem solving approaches, pictures and translations as needed.<br>○ Modify the linguistic complexity of tasks by rephrasing math problems.<br>○ Teacher models the thinking process used and the academic vocabulary needed to solve multistep problems that require students to interpret units consistently and accurately.<br>○ The students can explain through a “think aloud” and demonstrate how they solved the problem or explain their work.<br>○ Have students work in triads or small groups where they are able to support each other’s learning by giving each other input and filling in gaps in background. |

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| their work to describe problems given.<br>○ Compare and contrast the geometric and algebraic methods for performing the various vector operations.<br>○ Use a memory device to help differentiate terminology such as initial, terminal, dot, inner, and cross product. | |

**Enrichment**

- Extended learning goals:
  - ⇒ Students can calculate the resultant vector of three or more forces acting on an object (8.1 Enrichment: More than Two Forces Acting on an Object).
  - ⇒ Students can calculate linear combinations of vectors using the Linear Combination Theorem in v2 (8.2 Enrichment: Basis Vectors).
  - ⇒ Students can calculate linear combinations of vectors for three or more vectors (8.3 Enrichment: Basis Vectors in Three-Dimensional Space).
  - ⇒ Students can model vector functions using a table of values (8.4 Enrichment: Vector Equations).
  - ⇒ Students can determine whether two or three vectors are linearly dependent (8.5 Enrichment: Linearly Dependent Vectors).
  - ⇒ Students can apply parameter equations to determine the distance from a point to a given plane (8.6 Enrichment: Using Parameter Equations to Find the Distance from a Point to a Plane)
  - ⇒ Students can use a description to write the equation of a projectile fired at a given angle and initial velocity (8.7 Enrichment: Coordinate Equations of Projectiles).

| Unit 5: Polar Coordinates and Complex Numbers | |
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| **Desired Outcomes** | |
| **Established Goals: NJSLS**<br><br>○ Perform arithmetic operations with complex numbers (N.CN.3).<br>○ Represent complex numbers and their operations on the complex plane (N.CN.4, N.CN.5, N.CN.6).<br>○ Use complex numbers in polynomial identities and equations (N.CN.8).<br><br>**NJSLS Mathematical Practices**<br>1. Make sense of problems and persevere in solving them.<br>2. Reason abstractly and quantitatively.<br>3. Construct viable arguments and critique the reasoning of others.<br>4. Model with mathematics.<br>5. Use appropriate tools strategically.<br>6. Attend to precision.<br>7. Look for and make use of structure.<br>8. Look for and express regularity in repeated reasoning. | |
| **Enduring Understandings:**<br><br>○ The polar coordinate system can be used to display data in a different format from the Cartesian system.<br>○ Polar equations may be more useful in certain situations than rectangular equations. | **Essential Questions:**<br><br>○ How can we solve problems that involve a sphere (our world)?<br>○ How are complex numbers related to polar coordinates?<br>○ What is the relationship between the ordered pairs in the Cartesian coordinate system versus a polar system? |
| **Students will know:** | **Students will be able to:** |

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| ○ How to graph polar equations<br>○ Conversion between polar and rectangular coordinates<br>○ How to add, subtract, multiply and divide complex numbers in polar and rectangular form<br>○ How to convert between polar and rectangular forms of complex numbers<br>○ Find powers and roots of complex numbers<br>○ Key vocabulary: polar coordinate system, polar coordinates, pole, polar axis, polar equation, polar graph, rose, limacon, cardioid, classical curves, rectangular coordinates, iteration, complex conjugate, complex plane, Argand plane, absolute value, modulus, amplitude, argument, polar/trigonometric form. | ○ Graph points in polar coordinates.<br>○ Determine the distance between two points with polar coordinates.<br>○ Graph polar equations.<br>○ Convert between polar and rectangular coordinates.<br>○ Add, subtract, multiply, and divide complex numbers in rectangular form.<br>○ Graph complex numbers in the complex plane.<br>○ Convert complex numbers from rectangular to polar form and vice versa.<br>○ Find the product and quotient of complex numbers in polar form.<br>○ Find powers and roots of complex numbers in polar form using De Moivre's Theorem. |
| **Assessment Evidence** | |
| **<u>Suggested Performance Tasks:</u>**<br><br>○ Hangout PC Style: Students use graphing polar coordinates and equations, converting between polar and rectangular coordinates and equations, and identifying polar equations of conic sections to design a teen hangout. Have students:<br>a. design a dartboard on a polar grid consisting of two sections plus a bulls-eye with the radii and points earned labeled. Have students plot and label the location of two darts on their dartboards and find the distance between them.<br>b. design a room with a concert stage. Have | **<u>Required District/State Assessments:</u>**<br><br>○ Unit Assessment<br>○ SGO Assessments<br><br>**<u>Suggested Formative/Summative Assessments:</u>**<br><br>○ Describe Learning Vertically<br>○ Identify Key Building Blocks<br>○ Make Connections (between and among key building blocks)<br>○ Short/Extended Constructed Response Items<br>○ Multiple-Choice Items (where multiple answer choices may be correct) |

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| students represent the area of sound a microphone captures by writing and graphing an equation for a limaçon.<br>c. design a floor plan for a computer room on a complex plane. Computers and chairs should be labeled as complex numbers expressed in polar and rectangular forms. | ○ Drag and Drop Items<br>○ Use of Equation Editor<br>○ Quizzes<br>○ Journal Entries/Reflections/Quick-Writes<br>○ Accountable talk<br>○ Projects<br>○ Portfolio<br>○ Observation<br>○ Graphic Organizers/ Concept Mapping<br>○ Presentations<br>○ Role Playing<br>○ Teacher-Student and Student-Student Conferencing<br>○ Homework |

## Learning Plan

**Learning Activities:**

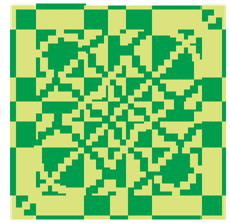
- Starter exercises
- Guided notes as necessary
- In class activities (QR codes, scavenger hunt, interactive exercises, exit ticket, etc.)

## Related Standards

**Interdisciplinary connections**

- Language Arts Connection (NJSLS.W.9-10.1.C)
  - ⇒ Example: ongoing throughout unit
- Architecture & Construction (9.3.12.AC.6)
  - ⇒ Example: Glencoe Advanced Math Concepts Textbook Page 566 #31.

**Textiles** Patterns in fabric can often be created by modifying a mathematical graph. The pattern at the right can be modeled by a lemniscate.



a. Suppose the designer wanted to begin with a lemniscate that was 6 units from end to end. What polar equation could have been used?

b. What polar equation could have been used to generate a lemniscate that was 8 units from end to end?

- Programming & Software Development (9.3.IT-PRG.4)
  - ⇒ Example: Glencoe Advanced Math Concepts Textbook Page 606 #38.

**Computer Graphics** Computer programmers can use complex numbers and the complex plane to implement geometric transformations. If a programmer starts with a square with vertices at $(2, 2)$, $(-2, 2)$, $(-2, -2)$, and $(2, -2)$, each of the vertices can be stored as a complex number in polar form. Complex number multiplication can be used to rotate the square $45°$ counterclockwise and dilate it so that the new vertices lie at the midpoints of the sides of the original square.

a. What complex number should the programmer multiply by to produce this transformation?

b. What happens if the original vertices are multiplied by the square of your answer to part a?

**<u>Technology (NJSLS Career Readiness, Life Literacies, and Key Skills)</u>**

- 9.4.12.TL.2: Generate data using formula-based calculations in a spreadsheet and draw conclusions about the data.
  - ⇒ Students will explore expressing, calculating the size, and operations with complex numbers in Google Sheets.

**<u>21st Century Skills (NJSLS Career Readiness, Life Literacies, and Key Skills)</u>**

- 9.4.12.IML.3: Analyze data using tools and models to make valid and reliable claims, or to determine optimal

design solutions (e.g., S-ID.B.6a., 8.1.12.DA.5, 7.1.IH.IPRET.8)
  - ⇒ Example: Students use technology such as graphing calculators and Desmos to graph polar equations on the polar plane and rectangular plane.

**NJ SEL Competencies**

- Self Management: Recognize the skills needed to establish and achieve personal and educational goals
- Social Awareness: Demonstrate an awareness of the expectations for social interactions in a variety of settings

## Culturally Relevant Connections

- Integrate Relevant Word Problems: Contextualize equations using word problems that reference student interests and cultures. Example: When interpreting functions, problems that relate to student interests such as music, sports and art enable the students to understand and relate to the concept in a more meaningful way.
- Encourage Student Leadership: Create an avenue for students to propose problem solving strategies and potential projects. Example: Students can learn to construct and compare polar and rectangular graphs by creating problems together and deciding if the problems fit the necessary criteria. This experience will allow students to discuss and explore their current level of understanding by applying the concepts to relevant real-life experiences.
- Present New Concepts Using Student Vocabulary: Use student diction to capture attention and build understanding before using academic terms. Example: Teach math vocabulary in various modalities for students to remember. Use multi-modal activities, analogies, realia, visual cues, graphic representations, gestures, pictures and cognates. Directly explain and model the idea of vocabulary words having multiple meanings. Students can create the Word Wall with their definitions and examples to foster ownership.
- Everyone has a Voice: Create a classroom environment where students know that their contributions are expected and valued. Example: Norms for sharing are established that communicate a growth mindset for mathematics. All students are capable of expressing mathematical thinking and contributing to the classroom community. Students learn new ways of looking at problem solving by working with and listening to each other.

## Accommodations

| **<u>Special Education/ 504/ At Risk Students Accommodations & Modifications:</u>** | **<u>ELL:</u>** |
| --- | --- |
| ○ Anchor charts to model strategies<br>○ Review Algebra concepts to ensure students have the information needed to progress in understanding<br>○ Provide reference sheets that list formulas, step by step procedures, theorems, and modeling of strategies<br>○ Teacher modeling of thinking processes<br>○ Introduce concepts embedded in real-life to help students relate to the mathematics involved such as reading a world map which uses polar coordinates.<br>○ Utilize technology through interactive sites to represent nonlinear data<br>○ Use videos to reinforce skills and thinking behind concepts.<br>○ Highlight and label solution steps for multi-step problems in different colors.<br>○ Encourage students to verbalize their thinking while working in small groups by asking, assessing, and advancing questions.<br>○ Allow time for questions to check on student understanding.<br>○ Compare and contrast distance formula in Algebra to polar coordinates.<br>○ Write steps detailing the processes for converting x=y to polar form.<br>○ Assist students in writing theorems in their own words.<br>○ When having difficulty drawing polar graphs, suggest the wheel and spoke graphing method. | ○ Translation dictionary<br>○ Sentence stems<br>○ Pre-teach pertinent vocabulary<br>○ Word wall with visual representations of mathematical terms<br>○ Graphic organizers to help students interpret the meaning in an expression or equation in context<br>○ Use anchor charts with important terms, problem solving approaches, pictures and translations as needed.<br>○ Modify the linguistic complexity of tasks by rephrasing math problems.<br>○ Teacher models the thinking process used and the academic vocabulary needed to solve multistep problems that require students to interpret units consistently and accurately.<br>○ The students can explain through a “think aloud” and demonstrate how they solved the problem or explain their work.<br>○ Have students work in triads or small groups where they are able to support each other’s learning by giving each other input and filling in gaps in background. |

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## Enrichment

- Extended learning goals:
  - ⇒ Students can use polar coordinates and the circumference of the earth to determine distances between points on the earth's surface (9.1 Enrichment: Distance on the Earth's Surface).
  - ⇒ Students can identify the symmetry of and graph polar equations on polar grid paper (9.2 Enrichment: Symmetry in Graphs of Polar Equations).
  - ⇒ Students can use polar equations to graph polar roses and use the graphs to generalize characteristics of polar roses (9.3 Enrichment: Polar Roses).
  - ⇒ Students can calculate the conjugates of complex numbers (9.7 Enrichment: Complex Conjugates).
  - ⇒ Students can show that complex numbers are algebraic by finding a polynomial with integer coefficients of which the given number is zero (9.8 Enrichment: Algebraic Numbers).

| Unit 6: Conic Sections | |
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| **Desired Outcomes** | |
| **Established Goals: NJSLS**<br><br>○ Translate between the geometric description and the equation for a conic section (G.GPE.3).<br><br>**NJSLS Mathematical Practices**<br>1. Make sense of problems and persevere in solving them.<br>2. Reason abstractly and quantitatively.<br>3. Construct viable arguments and critique the reasoning of others.<br>4. Model with mathematics.<br>5. Use appropriate tools strategically.<br>6. Attend to precision.<br>7. Look for and make use of structure.<br>8. Look for and express regularity in repeated reasoning. | |
| **Enduring Understandings:**<br><br>○ Conic sections are formed when a plane intersects a cone.<br>○ Conic sections reflect real-world phenomena. | **Essential Questions:**<br><br>○ Where do conic sections appear in the real world?<br>○ How does the intersection of a plane and a cone create the different conic sections?<br>○ How is an equation used to determine the type of conic? |
| **Students will know:**<br><br>○ The type of conic can be determined by the standard form of an equation.<br>○ Key vocabulary: conic section, degenerate conic, | **Students will be able to:**<br><br>○ Use and determine the standard and general forms of the equation of a circle, ellipse, hyperbola, and parabola. |

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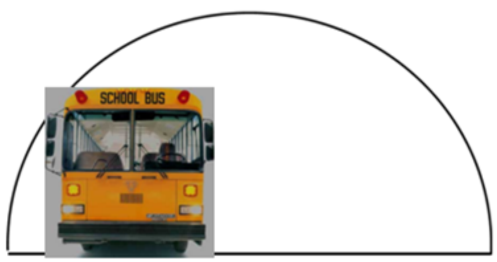
| locus, parabola, focus, directrix, axis of symmetry, vertex, ellipse, foci, major axis, center, minor axis, vertices, co-vertices, eccentricity, hyperbola, transverse axis, conjugate axis | ○ Graph circles, ellipses, hyperbolas, and parabolas. |

## Assessment Evidence

**Suggested Performance Tasks:**

- A train bridge is supported by a rectangular hyperbolic arch (a=b). The base of the arch is 120m wide, and the vertex is 20m above the base.
    a) Determine an equation of the hyperbola by letting the vertex have coordinates (0,-a).
    b) Determine the height of the arch above the base at a point 30 m from the center of the base.
- A tunnel is in the shape of a semi-ellipse 8m wide and 3.5m high. If a school bus is 2.44m wide, what is the maximum possible side height of the bus if it is to pass through the tunnel by:
    a) Staying in its lane?
    b) Straddling the centerline?




**Required District/State Assessments:**

- Unit Assessment
- SGO Assessments

**Suggested Formative/Summative Assessments:**

- Describe Learning Vertically
- Identify Key Building Blocks
- Make Connections (between and among key building blocks)
- Short/Extended Constructed Response Items
- Multiple-Choice Items (where multiple answer choices may be correct)
- Drag and Drop Items
- Use of Equation Editor
- Quizzes
- Journal Entries/Reflections/Quick-Writes
- Accountable talk
- Projects
- Portfolio
- Observation
- Graphic Organizers/ Concept Mapping
- Presentations
- Role Playing
- Teacher-Student and Student-Student

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| | Conferencing<br>○ Homework |

## Learning Plan

**Learning Activities:**

- Starter exercises
- Guided notes as necessary
- In class activities (QR codes, scavenger hunt, interactive exercises, exit ticket, etc.)

## Related Standards

**Interdisciplinary connections**

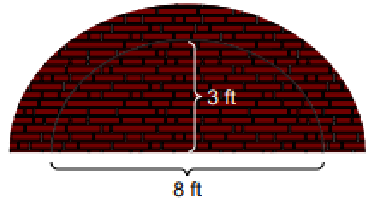
- Language Arts Connection (NJSLS.W.9-10.1.C)
  - ⇒ Example: ongoing throughout unit
- Marketing (9.3.MK-MGT.5)
  - ⇒ Example: Glencoe Advanced Math Concepts Textbook Page 630 #57.

    **Business** A pharmaceutical company manufactures two drugs. Each case of drug A requires 3 hours of processing time and 1 hour of curing time per week. Each case of drug B requires 5 hours of processing time and 5 hours of curing time per week. The schedule allows 55 hours of processing time and 45 hours of curing time weekly. The company must produce no more than 10 cases of drug A and no more than 9 cases of drug B. *(Lesson 2-7)*

    **a.** If the company makes a profit of $320 on each case of drug A and $500 on each case of drug B, how many cases of each drug should be produced in order to maximize profit?

    **b.** Find the maximum profit.
- Architecture & Construction (9.3.12.AC.6)
  - ⇒ Example: Glencoe Advanced Math Concepts Textbook Page 640 #52.

**Construction** The arch of a fireplace is to be constructed in the shape of a semi-ellipse. The opening is to have a height of 3 feet at the center and a width of 8 feet along the base. To sketch the outline of the fireplace, the contractor uses an 8-foot string tied to two thumbtacks.

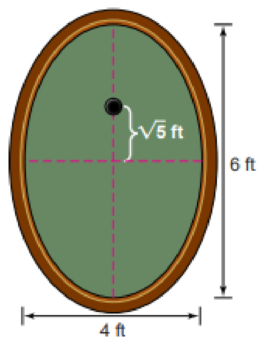



**a.** Where should the thumbtacks be placed?

**b.** Explain why this technique works.

- Recreation, Amusement & Attractions (9.3.HT-REC.3)
  - ⇒ Example: Glencoe Advanced Math Concepts Textbook Page 639 #47.

**Entertainment** *Elliptipool* is a billiards game that use an elliptically-shaped pool table with only one pocket in the surface. A cue ball and a target ball are used in play. The object of the game is to strike the target ball with the cue ball so that the target ball rolls into the pocket after one bounce off the side. Suppose the cue ball and target ball can be placed anywhere on the half of the table opposite the pocket. The pool table shown at the right is 4 feet wide and 6 feet long. The pocket is located $\sqrt{5}$ feet from the center of the table along the ellipse's major axis. Assuming no spin is placed on either ball and the target ball is struck squarely, where should each be placed to have the best chance of hitting the target ball into the pocket? Explain your reasoning.




## Technology (NJSLS Career Readiness, Life Literacies, and Key Skills)

- 9.4.12.TL.1: Assess digital tools based on features such as accessibility options, capacities, and utility for accomplishing a specific task (e.g., W.11-12.6.).
  - ⇒ Example: Students will use digital tools such as TI-84, Desmos, Google Suite, etc. to access, manage, evaluate, and synthesize information in order to solve problems individually and collaboratively to create and communicate knowledge.

## 21st Century Skills (NJSLS Career Readiness, Life Literacies, and Key Skills)

- 9.4.12.IML.3: Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions.
  - ⇒ Example: Students use the Desmos platform to complete activities such as "Building Conic Sections" and "Creative Conics."
- 9.4.12.CT.1: Identify problem-solving strategies used in the development of an innovative product or practice.
  - ⇒ Example: Students will work collaboratively in groups to solve mathematical tasks. Students will listen to or read the arguments of others and ask probing questions to clarify or improve arguments. Have teams of students work through the analytical proofs on conics. Then have each team explain its procedure to the class. Teams should show all necessary diagrams and equations.

## NJ SEL Competencies

- Responsible Decision-Making: Identify the consequences associated with one's actions in order to make constructive choices
- Relationship Skills: Utilize positive communication and social skills to interact effectively with others

## Climate Change

- NUCLEAR ENERGY Students will apply hyperbolas to model the nuclear cooling tower, or hyperboloid. (Glencoe Advanced Math Concepts 2006 Textbook p. 651 #45)

**Nuclear Power** A nuclear cooling tower is a *hyperboloid*, that is, a hyperbola rotated around its conjugate axis. Suppose the hyperbola used to generate the hyperboloid modeling the shape of the cooling tower has an eccentricity of $\frac{5}{3}$.

**a.** If the cooling tower is 150 feet wide at its narrowest point, determine an equation of the hyperbola used to generate the hyperboloid.

**b.** If the tower is 450 feet tall, the top is 100 feet above the center of the hyperbola, and the base is 350 feet below the center, what is the radius of the top and the base of the tower?

- ENERGY  Students will apply parabolas to calculate the focal length and depth of a mirror designed to harness the sun's rays to power a furnace on a tower. (Glencoe Advanced Math Concepts 2006 Textbook p. 653-55 Intro & Example 2)

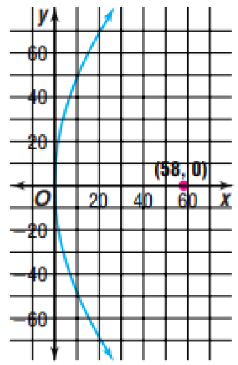
**Example 2** **ENERGY** **Refer to the application at the beginning of the lesson.**

Real World Application

**a. Find and graph the equation of a parabola that models the shape of the Odeillo mirror.**

**b. Find the depth of the parabolic mirror.**

**a.** The shape of the mirror can be modeled by a parabola with vertex at the origin and opening to the right. The general equation of such a parabola is $y^2 = 4px$, where $p$ is the focal length. Given a focal length of 58 feet, we can derive the model equation.




$y^2 = 4px$

$y^2 = 4(58)x \quad p = 58$

$y^2 = 232x$

**b.** With the mirror's vertex at the origin, the distance from the vertex to one edge of the mirror is half the overall width of the mirror, $\frac{1}{2}$(138 feet) or 69 feet.

Use the model equation to find the depth $x$ of the mirror when the distance from the center is 69 feet.

$y^2 = 232x$

$(69)^2 = 232x \quad y = 69$

$4761 = 232x$

$x = \frac{4761}{232}$ or about 20.5

The mirror is about 20.5 feet deep.

## Culturally Relevant Connections

- *Integrate Relevant Word Problems*: Contextualize equations using word problems that reference student interests and cultures. Example: When interpreting functions, problems that relate to student interests such as music, sports and art enable the students to understand and relate to the concept in a more meaningful way.

- *Encourage Student Leadership*: Create an avenue for students to propose problem solving strategies and potential projects. Example: Students can learn to construct and compare conic sections graphs by creating problems together and deciding if the problems fit the necessary criteria. This experience will allow students to discuss and explore their current level of understanding by applying the concepts to relevant real-life experiences.
- *Present New Concepts Using Student Vocabulary*: Use student diction to capture attention and build understanding before using academic terms. Example: Teach math vocabulary in various modalities for students to remember. Use multi-modal activities, analogies, realia, visual cues, graphic representations, gestures, pictures and cognates. Directly explain and model the idea of vocabulary words having multiple meanings. Students can create the Word Wall with their definitions and examples to foster ownership.
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## Accommodations

| **Special Education/ 504/ At Risk Students Accommodations & Modifications:** | **ELL:** |
| --- | --- |
| ○ Anchor charts to model strategies<br>○ Review Algebra concepts to ensure students have the information needed to progress in understanding<br>○ Provide reference sheets that list formulas, step by step procedures, theorems, and modeling of strategies<br>○ Teacher modeling of thinking processes<br>○ Introduce concepts embedded in real-life to help | ○ Translation dictionary<br>○ Sentence stems<br>○ Pre-teach pertinent vocabulary<br>○ Word wall with visual representations of mathematical terms<br>○ Graphic organizers to help students interpret the meaning in an expression or equation in context<br>○ Use anchor charts with important terms, problem solving approaches, pictures and translations as needed. |

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| students relate to the mathematics involved<br>○ Utilize technology through interactive sites to represent nonlinear data<br>○ Use videos to reinforce skills and thinking behind concepts.<br>○ Highlight and label solution steps for multi-step problems in different colors.<br>○ Encourage students to verbalize their thinking while working in small groups by asking, assessing, and advancing questions.<br>○ Have students find the first five terms of a sequence to see a pattern in order to understand the meaning of $(-1)^n$<br>○ Use real world problems to help students relate to arithmetic and geometric sequences.<br>○ Allow time for questions to check on student understanding. | ○ Modify the linguistic complexity of tasks by rephrasing math problems.<br>○ Teacher models the thinking process used and the academic vocabulary needed to solve multistep problems that require students to interpret units consistently and accurately.<br>○ The students can explain through a “think aloud” and demonstrate how they solved the problem or explain their work.<br>○ Have students work in triads or small groups where they are able to support each other’s learning by giving each other input and filling in gaps in background. |

**Enrichment**

- Extended learning goals:
  - ⇒ Students can apply the equation of a sphere to great circles and use completing the square to determine the center and radius of a sphere (10.2 Enrichment: Spheres).
  - ⇒ Students can graph and identify the characteristics of superellipses (10.3 Enrichment: Superellipses).
  - ⇒ Students can identify the characteristics of foci as the values of a, b, and c change (10.4 Enrichment: Moving Foci).

| Unit 7: Sequences and Series | |
| --- | --- |
| Desired Outcomes | |
| **Established Goals: NJSLS**<br><br>◦ Write expressions in equivalent forms to solve problems (A.SSE.4).<br>◦ Use polynomial identities to solve problems (A.APR.5).<br>◦ Build a function that models a relationship between two quantities (F.BF.2).<br><br>**NJSLS Mathematical Practices**<br>1. Make sense of problems and persevere in solving them.<br>2. Reason abstractly and quantitatively.<br>3. Construct viable arguments and critique the reasoning of others.<br>4. Model with mathematics.<br>5. Use appropriate tools strategically.<br>6. Attend to precision.<br>7. Look for and make use of structure.<br>8. Look for and express regularity in repeated reasoning. | |
| **Enduring Understandings:**<br><br>◦ Various methods can be used to evaluate sequences and series.<br>◦ Patterns of change are related to the behaviors of functions.<br>◦ Mathematical patterns can help simplify complex situations. | **Essential Questions:**<br><br>◦ Why is it helpful to know more than one way to evaluate sequences and series?<br>◦ How are patterns of change related to the behaviors of functions?<br>◦ How are mathematical patterns used to simplify complex situations? |
| **Students will know:** | **Students will be able to:** |

- Formulas that relate to geometric and arithmetic functions.
- Sequences and series are related to functions.
- Key vocabulary: sequence, terms, arithmetic sequence, common difference, recursive formula, arithmetic means, arithmetic series, nth partial sum, geometric sequence, common ratio, geometric means, geometric series, infinite sequence, limit, infinite series, sigma notation, index of summation, n factorial

- Find the nth term and arithmetic/geometric means of an arithmetic/geometric sequence.
- Find the sum of n terms of an arithmetic and geometric series.
- Find the limit of the terms of an infinite sequence.
- Find the sum of an infinite geometric series.
- Use sigma notation.

## Assessment Evidence

**Suggested Performance Tasks:**

Real-life series - Choose real-life situations which use arithmetic or geometric sequences and series. You must have at least one of each type for this project.

- Research- Include any research you did to discover the real-life applications of sequences and series. If you created the real-life applications yourself, explain your thinking. If you used ideas from other sources, show how you changed the terms, common difference, or common ratio to make your application unique.
- Diagrams or pictures - Include a diagram or picture of the situations you have chosen. Either write out the 1st several terms, or use pictures to represent what is taking place. For example, if a ball is bouncing you might want to show the distance traveled in the 1st several bounces.
- Formulas - Write the recursive and explicit formulas for each sequence in the series. Then

**Required District/State Assessments:**

- Unit Assessment
- SGO Assessments

**Suggested Formative/Summative Assessments:**

- Describe Learning Vertically
- Identify Key Building Blocks
- Make Connections (between and among key building blocks)
- Short/Extended Constructed Response Items
- Multiple-Choice Items (where multiple answer choices may be correct)
- Drag and Drop Items
- Use of Equation Editor
- Quizzes
- Journal Entries/Reflections/Quick-Writes
- Accountable talk
- Projects

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| write the series using summation notation.<br>○ Show what you know - Use as many of the concepts about arithmetic and geometric sequences and series as you can to describe your real-life situations. For example, show how to find the nth term of your arithmetic sequence or describe how to evaluate the 1st n terms. Discuss whether your geometric series is finite or infinite and how you know. Pretend you have only 2 terms in the sequence. Describe how you could write a rule for the nth term. Make up your own questions about your sequences and series and then answer them yourself. | ○ Portfolio<br>○ Observation<br>○ Graphic Organizers/ Concept Mapping<br>○ Presentations<br>○ Role Playing<br>○ Teacher-Student and Student-Student Conferencing<br>○ Homework |

## Learning Plan

**Learning Activities:**

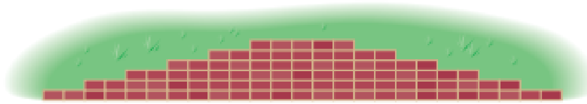
- Starter exercises
- Guided notes as necessary
- In class activities (QR codes, scavenger hunt, interactive exercises, exit ticket, etc.)

## Related Standards

**Interdisciplinary connections**

- Language Arts Connection (NJSLS.W.9-10.1.C)
  - ⇒ Example: ongoing throughout unit
- Architecture & Construction (9.3.12.AC.6)
  - ⇒ Example: Students can use arithmetic sequences to plan a tiling pattern project (Glencoe Advanced Math Concepts Textbook Page 764 #52)

**Construction** The Arroyos are planning to build a brick patio that approximates the shape of a trapezoid. The shorter base of the trapezoid needs to start with a row of 5 bricks, and each row must increase by 2 bricks on each side until there are 25 rows. How many bricks do the Arroyos need to buy?



- Manufacturing (9.3.MN.5)
  - ⇒ Example: Students can calculate the amount a business is charged by the Environmental Protection Agency if it does not pass inspection (Glencoe Advanced Math Concepts Textbook Page 795 Example 2)

**Example 2** **MANUFACTURING** **Refer to the application at the beginning of this lesson.**

Real World Application

**a. How much is the company fined on the 20th day?**

**b. What is the total amount in fines owed by the manufacturing plant?**

**c. Represent this sum using sigma notation.**

- Science Connection (HS-PS1-8)
  - ⇒ Example: Students can calculate the colony of bacteria undergoing radioactive decay (Glencoe Advanced Math Concepts Textbook Page 772 #41)

**Biology** A cholera bacterium divides every half-hour to produce two complete cholera bacteria.

a. If an initial colony contains a population of $b_0$ bacteria, write an equation that will determine the number of bacteria present after $t$ hours.

b. Suppose a petri dish contains 30 cholera bacteria. Use the equation from part **a** to determine the number of bacteria present 5 hours later.

c. What assumptions are made in using the formula found in part **a**?

**Technology (NJSLS Career Readiness, Life Literacies, and Key Skills)**

- 9.4.12.TL.2: Generate data using formula-based calculations in a spreadsheet and draw conclusions about the data.
  - ⇒ Example: Students will create sequence functions in Google Sheets to solve application problems.

## 21st Century Skills (NJSLS Career Readiness, Life Literacies, and Key Skills)

- 9.4.12.IML.3: Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions.
    - ⇒ Example: Students use graphing calculators to evaluate and create series in sigma notation.
- 9.1.12.PB.6: Describe and calculate interest and fees that are applied to various forms of spending, debt and saving.
    - ⇒ Example: Students can use compound interest formulas to calculate the amount of money in an account after t amount of time (Glencoe Advanced Math Concepts Textbook Page 770 Example 6).

**INVESTMENTS** **Hiroshi wants to begin saving money for college. He decides to deposit $500 at the beginning of each quarter (January 1, April 1, July 1, and October 1) in a savings account that pays an APR of 6% compounded quarterly. The interest for each quarter is posted on the last day of the quarter. Determine Hiroshi's account balance at the end of one year.**

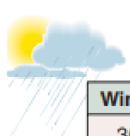
## NJ SEL Competencies

- Self-Awareness: Recognize the importance of self-confidence in handling daily tasks and challenges
- Self-Management: Recognize the skills needed to establish and achieve personal and educational goals

## Climate Change

- TEMPERATURES Students will apply sinusoidal functions to model temperatures patterns (Glencoe Advanced Math Concepts 2006 Textbook p. 773 #55).

**Weather** The maximum normal daily temperatures in each season for Lincoln, Nebraska, are given below. Write a sinusoidal function that models the temperatures, using $t = 1$ to represent winter. *(Lesson 6-6)*



**Normal Daily Temperatures for Lincoln, Nebraska**

| Winter | Spring | Summer | Fall |
|---|---|---|---|
| 36° | 61° | 86° | 65° |

Source: Rand McNally & Company

- POPULATION  Students will utilize census data to write an arithmetic and geometric sequence formulas to model the population and predict future populations.  Students will write a one-page paper comparing the models and discussing which formula best models the data. (Glencoe Advanced Math Concepts 2006 Textbook p. 833)
    a) Use the internet to find the population of the United States from at least 1900 through 2000. Write a sequence using the population for each ten-year interval, for example, 1900, 1910, and so on.
    b) Write a formula for an arithmetic sequence that provides a reasonable model for the population sequence.
    c) Write a formula for a geometric sequence that provides a reasonable model for the population sequence.
    d) Use your models to predict the US population for the year 2050.  Write a one-page paper comparing the arithmetic and geometric sequences you used to model the population data. Discuss which formula you think best models the data.

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- Utilize technology through interactive sites to represent nonlinear data
- Use videos to reinforce skills and thinking behind concepts.
- Highlight and label solution steps for multi-step problems in different colors.
- Encourage students to verbalize their thinking while working in small groups by asking, assessing, and advancing questions.
- Emphasize that the difference between any two terms must be the same for the sequence to be arithmetic.
- Suggest finding the missing terms by working back from the last term given.

<u>ELL:</u>

- Translation dictionary
- Sentence stems
- Pre-teach pertinent vocabulary
- Word wall with visual representations of mathematical terms
- Graphic organizers to help students interpret the meaning in an expression or equation in context
- Use anchor charts with important terms, problem solving approaches, pictures and translations as needed.
- Modify the linguistic complexity of tasks by rephrasing math problems.
- Teacher models the thinking process used and the academic vocabulary needed to solve multistep problems that require students to interpret units consistently and accurately.
- The students can explain through a “think aloud” and demonstrate how they solved the problem or explain their work.
- Have students work in triads or small groups where they are able to support each other’s learning by giving each other input and filling in gaps in background.

| | |
|---|---|
| ○ Allow time for questions to check on student understanding. | |

| **Enrichment** |
|---|
| ○ Extended learning goals:<br>⇒ Students can determine quadratic expressions for sequences using the finite differences method (12.1 Enrichment: Quadratic Formulas for Sequences).<br>⇒ Students can describe and graph geometric sequences (12.2 Enrichment: Basis Vectors).<br>⇒ Students can use limits of sequences to solve quadratic equations (12.3 Enrichment: Solve Equations using Sequences). |

# Appendix A: Culturally Relevant Pedagogy Examples

## BUILDING EQUITY IN YOUR TEACHING PRACTICE

How do the essential questions highlight the connection between the big ideas of the unit and equity in your teaching practice?

| CONTENT INTEGRATION | KNOWLEDGE CONSTRUCTION | PREJUDICE REDUCTION | EQUITABLE PEDAGOGY | EMPOWERING SCHOOL CULTURE |
|---|---|---|---|---|
| Teachers use examples and content from a variety of cultures & groups. | Teachers help students understand how knowledge is created and influenced by cultural assumptions, perspectives & biases. | Teachers implement lessons and activities to assert positive images of ethnic groups & improve intergroup relations. | Teachers modify techniques and methods to facilitate the academic achievement of students from diverse backgrounds. | Using the other four dimensions to create a safe and healthy educational environment for all. |
| This unit / lesson is connected to other topics explored with students.<br><br>There are multiple viewpoints reflected in the content of this unit / lesson.<br><br>The materials and resources are reflective of the diverse identities and experiences of students.<br><br>The content affirms students, as well as exposes them to experiences other than their own. | This unit / lesson provides context to the history of privilege and oppression.<br><br>This unit / lesson addresses power relationships.<br><br>This unit / lesson help students to develop research and critical thinking skills.<br><br>This curriculum creates windows and mirrors* for students. | This unit / lesson help students question and unpack biases & stereotypes.<br><br>This unit / lesson help students examine, research and question information and sources.<br><br>The curriculum encourage discussion and understanding about the groups of people being represented.<br><br>This unit / lesson challenges dominant perspectives. | The instruction has been modified to meet the needs of each student.<br><br>Students feel respected and their cultural identities are valued.<br><br>Additional supports have been provided for students to become successful and independent learners.<br><br>Opportunities are provided for student to reflect on their learning and provide feedback. | There are opportunities for students to connect with the community.<br><br>My classroom is welcoming and supportive for all students?<br><br>I am aware of and sensitive to the needs of my students and their families.<br><br>There are effective parent communication systems established. Parents can talk to me about issues as they arise in my classroom. |

Developed by Karla E. Vigil. Adapted with permission from James A. Banks, CULTURAL DIVERSITY AND EDUCATION: FOUNDATIONS, CURRICULUM, AND TEACHING (6th edition), New York: Routledge, 2016, page 5 and Gordon School Institute on Multicultural Practice.

# Appendix B: English Language Learners

WIDA Levels:

At the given level of English language proficiency, English language learners will process, understand, produce or use

| | |
|---|---|
| **6- Reaching** | • Specialized or technical language reflective of the content areas at grade level<br>• A variety of sentence lengths of varying linguistic complexity in extended oral or written discourse as required by the specified grade level<br>• Oral or written communication in English comparable to proficient English peers |
| **5- Bridging** | • Specialized or technical language of the content areas<br>• A variety of sentence lengths of varying linguistic complexity in extended oral or written discourse, including stories, essays or reports<br>• Oral or written language approaching comparability to that of proficient English peers when presented with grade level material. |
| **4- Expanding** | • Specific and some technical language of the content areas<br>• A variety of sentence lengths of varying linguistic complexity in oral discourse or multiple, related sentences or paragraphs<br>• Oral or written language with minimal phonological, syntactic or semantic errors that may impede the communication, but retain much of its meaning, when presented with oral or written connected discourse, with sensory, graphic or interactive support |
| **3- Developing** | • General and some specific language of the content areas<br>• Expanded sentences in oral interaction or written paragraphs<br>• Oral or written language with phonological, syntactic or semantic errors that may impede the communication, but retain much of its meaning, when presented with oral or written, narrative or expository descriptions with sensory, graphic or interactive support |
| **2- Beginning** | • General language related to the content area<br>• Phrases or short sentences<br>• Oral or written language with phonological, syntactic, or semantic errors that often impede of the communication when presented with one to multiple-step commands, directions, or a series of statements with sensory, graphic or interactive support |
| **1- Entering** | • Pictorial or graphic representation of the language of the content areas<br>• Words, phrases or chunks of language when presented with one-step commands directions, WH-, choice or yes/no questions, or statements with sensory, graphic or interactive support |

# Language Development Supports For English Language Learners To Increase Comprehension and Communication Skills

## Environment

- Welcoming and stress-free
- Respectful of linguistic and cultural diversity
- Honors students' background knowledge
- Sets clear and high expectations
- Includes routines and norms
- Is thinking-focused vs. answer-seeking
- Offers multiple modalities to engage in content learning and to demonstrate understanding
- Includes explicit instruction of specific language targets
- Provides participation techniques to include all learners
- Integrates learning centers and games in a meaningful way
- Provides opportunities to practice and refine receptive and productive skills in English as a new language
- Integrates meaning and purposeful tasks/activities that:
  - Are accessible by all students through multiple entry points
  - Are relevant to students' lives and cultural experiences
  - Build on prior mathematical learning
  - Demonstrate high cognitive demand
  - Offer multiple strategies for solutions
  - Allow for a language learning experience in addition to content

| Sensory Supports* | Graphic Supports* | Interactive Supports* | Verbal and Textual Supports |
|---|---|---|---|
| • Real-life objects (realia) or concrete objects<br>• Physical models<br>• Manipulatives<br>• Pictures & photographs<br>• Visual representations or models such as diagrams or drawings<br>• Videos & films<br>• Newspapers or magazines<br>• Gestures<br>• Physical movements<br>• Music & songs | • Graphs<br>• Charts<br>• Timelines<br>• Number lines<br>• Graphic organizers<br>• Graphing paper | • In a whole group<br>• In a small group<br>• With a partner such as *Turn-and-Talk*<br>• In pairs as a group (first, two pairs work independently, then they form a group of four)<br>• In triads<br>• Cooperative learning structures such as *Think-Pair-Share*<br>• Interactive websites or software<br>• With a mentor or coach | • Labeling<br>• Students' native language<br>• Modeling<br>• Repetitions<br>• Paraphrasing<br>• Summarizing<br>• Guiding questions<br>• Clarifying questions<br>• Probing questions<br>• Leveled questions such as *What? When? Where? How? Why?*<br>• Questioning prompts & cues<br>• Word Banks<br>• Sentence starters<br>• Sentence frames<br>• Discussion frames<br>• Talk moves, including *Wait Time* |

*from *Understanding the WIDA English Language Proficiency Standards. A Resource Guide.* 2007 Edition.. Board of Regents of the University of Wisconsin System, on behalf of the WIDA Consortium—www.wida.us.

*Galina (Halla) Jmourko, ESOL Coach, PGCPS; 2015, Rvsd. 2016*

# Appendix C: WIDA ELD Standards Integration

<table>
<tr><td>ELD-MA.9-12 Explain Interpretive</td><td>Interpret mathematical explanations by<ul><li>Identifying concept or entity</li><li>Analyzing data and owning problem-solving approaches</li><li>Evaluating rationales, models, and/or interpretations based on evidence and mathematical principles</li></ul></td></tr>
<tr><td>ELD-MA 9-12 Explain Expressive</td><td>Construct mathematical explanations that<ul><li>Introduce mathematical concept or entity</li><li>Share solutions with others</li><li>Describe data and/or approach used to solve a problem</li><li>State reasoning used to generate own or alternate solutions</li></ul></td></tr>
<tr><td>ELD-MA.9-12 Argue Interpretive</td><td>Interpret mathematics arguments by<ul><li>Comparing conjectures with previously established results and stated assumptions</li><li>Distinguishing correct from flawed logic</li><li>Evaluating relationships among evidence and mathematical principles to create generalizations</li></ul></td></tr>
<tr><td>ELD-MA.9-12 Argue Expressive</td><td>Construct mathematics arguments that<ul><li>Introduce mathematical concept or entity</li><li>Share solutions with others</li><li>Describe data and/or approach used to solve a problem</li><li>State reasoning used to generate own or alternate solutions</li></ul></td></tr>
</table>

# Appendix D: Differentiated Instruction

Strategies to accommodate based on student individual needs::

1. Time/General
   a. Extra time for assigned tasks
   b. Adjust length of assignment
   c. Timeline with due dates for reports and projects
   d. Communication system between home and school
   e. Provide lecture notes/outline

2. Processing
   a. Extra Response time
   b. Have students verbalize steps
   c. Repeat, clarify or reword directions
   d. Mini-breaks between tasks
   e. Provide a warning for transitions
   f. Partnering

3. Comprehension
   a. Precise processes for balanced math instructional model
   b. Short manageable tasks
   c. Brief and concrete directions
   d. Provide immediate feedback
   e. Small group instruction
   f. Emphasize multi-sensory learning

4. Recall
   a. Teacher-made checklist
   b. Use visual graphic organizers
   c. Reference resources to
   d. promote independence
   e. Visual and verbal reminders
   f. Graphic organizers

5. Assistive Technology
   a. Computer/whiteboard
   b. Tape recorder
   c. Video Tape

6. Tests/Quizzes/Grading
   a. Extended time
   b. Study guides
   c. Shortened tests
   d. Read directions aloud

7. Behavior/Attention
   a. Consistent daily structured routine
   b. Simple and clear classroom rules
   c. Frequent feedback

8. Organization
   a. Individual daily planner
   b. Display a written agenda
   c. Note-taking assistance
   d. Color code materials

# Appendix E: Enrichment

What is the purpose of enrichment?

The purpose of enrichment is to provide extended learning opportunities and challenges to students who have already mastered, or can quickly master, the basic curriculum. Enrichment gives the student more time to study concepts with greater depth, breadth, and complexity.

- Enrichment also provides opportunities for students to pursue learning in their own areas of interest and strengths.
- Enrichment keeps advanced students engaged and supports their accelerated academic needs.
- Enrichment provides the most appropriate answer to the question, “What do you do when the student already knows it?”

| **Enrichment is ...** | **Enrichment is not...** |
|---|---|
| • Planned and purposeful<br>• Different, or differentiated, work – not just more work<br>• Responsive to students’ needs and situations<br>• A promotion of high-level thinking skills and making connections within content<br>• The ability to apply different or multiple strategies to the content<br>• The ability to synthesize concepts and make real world and cross curricular connections<br>• Elevated contextual complexity<br>• Sometimes independent activities, sometimes direct instruction<br>• Inquiry based or open-ended assignments and projects<br>• Using supplementary materials in addition to the normal range of resources<br>• Choices for students<br>• Tiered/Multi-level activities with flexible groups (may change daily or weekly) | • Just for gifted students (some gifted students may need intervention in some areas just as some other students may need frequent enrichment)<br>• Worksheets that are more of the same (busywork)<br>• Random assignments, games, or puzzles not connected to the content areas or areas of student interest<br>• Extra homework<br>• A package that is the same for everyone<br>• Thinking skills taught in isolation<br>• Unstructured free time |

# Appendix F: Resources

**Textbook:**

Holliday, Cuevas, McClure, Carter, and Marks. Advanced Mathematical Concepts. Glencoe, 2001 (Medford) & 2006 (West)

# Appendix G: Climate Change Curriculum Statement

With the adoption of the 2020 New Jersey Student Learning Standards (NJSLS), New Jersey became the first state in the nation to include climate change across content areas. These standards are designed to prepare students to understand how and why climate change happens, the impact it has on our local and global communities and to act in informed and sustainable ways.

Districts are encouraged to utilize the NJSLS to develop interdisciplinary units focused on climate change that include authentic learning experiences, integrate a range of perspectives and are action oriented. While the 2016 NJSLS-English Language Arts (ELA) and Mathematics do not have specific climate change standards, districts may want to consider how they can design interdisciplinary climate change units that incorporate relevant ELA and mathematics standards.

Components of this are tagged throughout the curriculum as appropriate under the “Related Standards” section in each unit.